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A NOTE ON TENSIONLESS STRINGS IN M-THEORY

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ABSTRACT

In this article we examine the appearance of tensionless strings in M-Theory. We subsequently interpret these tensionless strings in a String Theory context. In particular, we examine tensionless strings appearing in M-Theory on S^1, M-Theory on $S^1/\mathbf{Z}_2$, and M-Theory on T^2; we then interpret the appearance of such strings in a String Theory context. Then we reverse this process and examine the appearance of some tensionless strings in String Theory. Subsequently we interpret these tensionless strings in a M-Theory context.

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1. Introduction

The theory formerly known as string theory has undergone quite a revolution as of late. In the past year and a half the five string theories have all been related so as to form a single String Theory. Older results [8] relating the two Type II string theories and the two Heterotic string theories [5] have been combined with newer results [9] relating the Type II and Heterotic string theories so as to relate the Type IIA, Type IIB, $E_8 \times E_8$ Heterotic, and $SO(32)$ Heterotic string theories. Also, as of late, the $SO(32)$ Heterotic string theory has been related to the Type I string theory [12] thus completing the "loop" and relating all five consistent string theories and forming a single String Theory. Furthermore, there has also been progress on yet another front, relating String Theory to M-Theory [10][12], an eleven-dimensional theory containing two-branes and five-branes [4][13]. In relating M-Theory to String Theory one finds that basically all properties of String Theory may be derived from M-Theory. One may derive these String Theory properties by relating M-Theory to any given string theory, then relating the given string theory to all the other string theories by way of the various "string-string dualities." In this way one may derive various properties of String Theory from M-Theory.

In this article we will employ some of the relations between M-Theory and String Theory to study the appearance of tensionless strings in M-Theory. We will subsequently interpret the appearance of these tensionless strings in a String Theory context. Also, we will reverse this process and examine the appearance of tensionless strings in various string theories and interpret these tensionless strings in terms of M-Theory. The purpose of this study is to try and better understand so-called "phenomena of the second kind" [14] in which a p-brane becomes tensionless[1].

In particular, we will first consider the appearance of a tensionless string in M-Theory on S^1. M-Theory on S^1 is equivalent to the Type IIA string theory [10]. Obviously the Type IIA string theory possess a one-brane. From a M-Theory point-of-view this one-brane arises from a M-Theory two-brane wrapping about S^1. The tension of the resultant one-brane is given by $T_{2,M}R_{11}$ [11], where $T_{2,M}$ is the tension of the two-brane and R_{11} the S^1 radius. Hence, such a one-brane becomes tensionless as $R_{11} \to 0$. We will examine this limit from a Type IIA perspective. Also, we will reverse this process and consider a

[1] Note, "phenomena of the second kind" in [14] referred only to strings becoming tensionless; however, the "spirit" of the term is maintained if we extend this to p-branes becoming tensionless.

tensionless string appearing in the Type IIA string theory then interpret this tensionless string from a M-Theory perspective. After this we will move on to M-Theory on $S^1/\mathbf{Z}_2$.

Witten and Horava proved [12] that M-Theory on $S^1/\mathbf{Z}_2$ is equivalent to the $E_8 \times E_8$ Heterotic string theory. The $E_8 \times E_8$ Heterotic string theory obviously possess a one-brane. From a M-Theory point-of-view this one-brane arises from a M-Theory two-brane wrapping around $S^1/\mathbf{Z}_2$. The tension of the resultant one-brane is given by $T_{2,M}R_{11}$, where here R_{11} is the radius of $S^1/\mathbf{Z}_2$. Again, in this case the string becomes tensionless in the limit $T_{2,M}R_{11} \to 0$. We will examine this limit from a Heterotic perspective. After this we reverse this situation and consider a tensionless string in the $E_8 \times E_8$ Heterotic string theory and interpret it from a M-Theory perspective. Finally, we consider M-Theory on T^2.

M-Theory on T^2, as was shown by Witten [10], is equivalent to the Type IIA string theory on S^1. The Type IIA string theory on S^1 obviously possess a one-brane; in fact, it possess two different one-branes. The first one-brane is the standard one-brane of the Type IIA string theory and the second one-brane arises from a two-brane of ten-dimensional Type IIA string theory wrapping about the S^1 factor. From a M-Theory perspective these one-branes have a similar origin. The M-Theory two-brane can wrap about either one of the S^1 factors of $T^2 = S^1 \times S^1$. Wrapping about the first S^1 leads to one of the one-branes and wrapping about the second S^1 leads to the other one-brane. If we denote the radius of the first S^1 as R_{10} and the radius of the second S^1 as R_{11}, then the tension of the first one-brane is $T_{2,M}R_{10}$ and the tension of the second one-brane is $T_{2,M}R_{11}$. So, in this situation we can obtain a tensionless one-brane by taking either R_{10} or R_{11} to zero. We will interpret these limits in terms of a Type II string theory on S^1. We then conclude with some general remarks on the various limits studied in this article.

2. Tensionless Strings : M-Theory on S^1

In this section we will examine a tensionless string appearing in M-Theory on S^1 from a Type IIA perspective. We will then examine a tensionless string in the Type IIA string theory from a M-Theory perspective. Let us start by examining the tensionless string in M-Theory on S^1 from a Type IIA perspective.

2.1. *Tensionless Strings in M-Theory on S^1*

In this subsection we will examine a tensionless string appearing in M-Theory on S^1 from a Type IIA perspective. Let us now start this examination.

M-Theory on S^1 is equivalent to the Type IIA string theory [10]. The Type IIA string theory obviously possess a one-brane. From a M-Theory perspective this one-brane arises from a two-brane wrapping about S^1. If we denote the radius of the S^1 as measured in the M-Theory metric as R_{11} and the tension of the two-brane as measured in the M-Theory metric as $T_{2,M}$, then the tension of this one-brane $T_{1,M}$ in the M-Theory metric is

$$T_{1,M} = T_{2,M} R_{11}. \tag{2.1}$$

So, one can see that if we wish the tension of the two-brane $T_{2,M}$ to remain constant, then taking the tension of the one-brane to zero entails taking the limit $R_{11} \rightarrow 0$. We will consider only this limit and not the limit in which $T_{2,M} \rightarrow 0$ as we are trying to understand the consequences of only a tensionless string and are not concerned at this point with tensionless two-branes. Also, we will assume that the M-Theory ten-metric $g_{10,M}$ behaves as $g_{10,M} \rightarrow g_{10,M}$ in this limit. So, in this limit the M-Theory target space takes the form seen in Figure 1A.

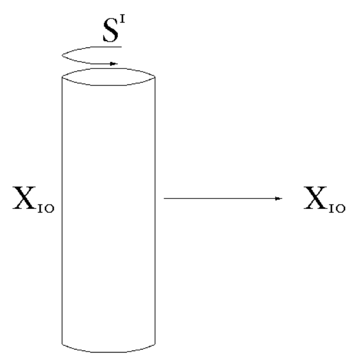

Figure 1A

where M-Theory is on $X_{10} \times S^1$ and X_{10} is an arbitrary ten-manifold. Now, let us consider this from a Type IIA perspective.

Employing the fact that M-Theory on S^1 is equivalent to the Type IIA string theory one can obtain various relations between the quantities defining the M-Theory compactification on S^1 and the parameters of the Type IIA string theory. In particular, one finds

[2][10] that the Type IIA ten-dimensional coupling constant $\lambda_{10,IIA}$ is related to the S^1 radius as measured in the M-Theory metric R_{11},

$$\lambda_{10,IIA} = R_{11}^{3/2}. \tag{2.2}$$

Furthermore, [2][10] one finds that the ten-dimensional M-Theory metric $g_{10,M}$ is related to the ten-dimensional Type IIA metric $G_{10,IIA}$ as

$$G_{10,IIA} = R_{11} g_{10,M}. \tag{2.3}$$

So, from this point-of-view one can see that the Type IIA string theory is singular in the limit $R_{11} \to 0$.

The metric of the ten-dimensional manifold, from a Type IIA string theory perspective, "vanishes." This is a result of the fact that we require the M-Theory metric $g_{10,M}$ of the ten-manifold X_{10} to be invariant in the limit $R_{11} \to 0$. From this Type IIA perspective the Type IIA target space takes the rather unfortunate singular form seen in Figure 1B.

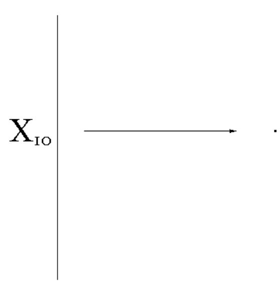

Figure 1B

Now, let us consider how in the limit $R_{11} \to 0$ the tensions of the M-Theory two-brane and wrapped one-brane appear in the Type IIA string theory.

Generically, a p-brane has a tension T_p with dimension (length)$^{-(p+1)}$. So, in particular, as the tension of a p-brane is a dimensionful quantity, it depends upon the metric in which it is measured. As the M-Theory ten-metric and the Type IIA ten-metric are scaled relative to one-another (2.3), the tension of any M-Theory p-brane is scaled relative to the tension of its Type IIA counterpart. Hence, if we write the tension of the two-brane as measured in the M-Theory ten-metric as $T_{2,M}$, then the tension of the two-brane $T_{2,IIA}$ in the Type IIA ten-metric is

$$T_{2,IIA} = R_{11}^{-3/2} T_{2,M}, \tag{2.4}$$

where the equality follows from (2.3). Employing similar logic we may calculate the tension of the one-brane in the Type IIA string theory. First though, remember that the one-brane arises from wrapping the M-Theory two-brane around S^1 and hence has a tension $T_{1,M}$ given by $T_{1,M} = T_{2,M}R_{11}$ in the M-Theory metric (2.1). Measuring this tension in the Type IIA metric one finds a tension $T_{1,IIA}$ given by,

$$T_{1,IIA} = R_{11}^{-1}T_{1,M} = T_{2,M}, \tag{2.5}$$

where the first equality follows from (2.3) and the second from (2.1).

Hence, in the limit $R_{11} \to 0$, the limit in which M-Theory obtains a tensionless string and a two-brane of finite tension, the Type IIA theory does not possess a tensionless string by way of equation (2.5), but the Type IIA theory does possess a two-brane of infinite tension, in accord with equation (2.4). In addition, according to equation (2.2), the Type IIA theory is weakly coupled in this region. As to what this all "means," I do not know.

However, what is certain is that for an arbitrary X_{10} we do not have a handle on the theory in this particular limit. From a Type IIA point-of-view, spacetime is becoming singular and in addition $T_{2,IIA} \to \infty$. One can see this in Figure 1B or equivalently in equation (2.3) as $R_{11} \to 0$ and $g_{10,M} \to g_{10,M}$. So, for general X_{10} we really do not know what is going on from a Type IIA perspective as we "generically" have no perturbative framework in which to work. However, in various special cases one may at least "resolve" the singular ten-manifold X_{10} from a Type IIA perspective. A particular example of this is if $X_{10} = T^{10}$. In this case one could perform a T-Duality transformation on each of the ten radii and obtain a Type IIA theory on $\mathbf{R}^{10}$. However, this is only a special case, and our understanding of this situation for a general X_{10} is minimal. So, let us move on and try to understand the tensionless strings of the Type IIA string theory from a M-Theory point-of-view.

2.2. Tensionless Strings in Type IIA String Theory

As we found in the previous subsection, the limit in which the one-brane of M-Theory on S^1 becomes tensionless is "generically" rather ill behaved. In this section we will reverse the process of last section. We will consider the appearance of a tensionless string in the Type IIA string theory, then we will interpret the appearance of this string in a M-Theory context. Most of the "leg-work" for this investigation was done in the previous subsection; so, we will rely heavily upon the previous subsection in our calculations.

Let us consider the limit in which the tension of the Type IIA one-brane vanishes $T_{1,IIA} \to 0$. Furthermore, let us also assume that $G_{10,IIA}$ and $T_{2,IIA}$ do not vary in this limit, $G_{10,IIA} \to G_{10,IIA}$ and $T_{2,IIA} \to T_{2,IIA}$. Graphically, this is represented in Figure 2A.

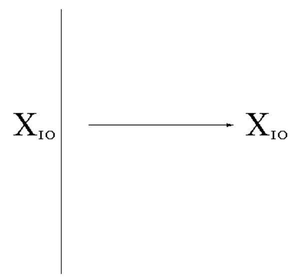

Figure 2A

As $T_{1,IIA} = T_{2,M}$ according to equation (2.5), one can see that in the limit $T_{1,IIA} \to 0$ the M-Theory two-brane tension $T_{2,M}$ goes to zero. Similarly, as $T_{2,IIA} = R_{11}^{-3/2} T_{2,M}$ in accord with equation (2.4), $R_{11} \to 0$ in this limit due to the fact that $T_{2,M} \to 0$ and $T_{2,IIA} \to T_{2,IIA}$. Furthermore, as the Type IIA ten-metric and the M-Theory ten-metric are related as in equation (2.3), the limit implies that $g_{10,M}$ becomes "flat." Graphically, this is depicted in Figure 2B

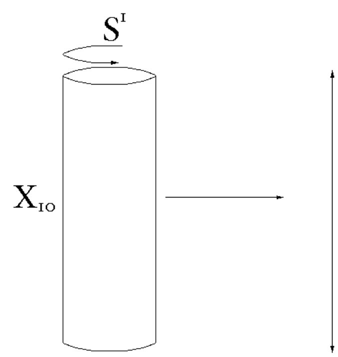

Figure 2B

where the arrowheads denote the fact that the ten-manifold from a M-Theory point-of-view is infinitely "stretched."

In this limit, for general X_{10}, M-Theory is singular. One can see this relatively easily from Figure 2B or equivalently from the fact that $R_{11} \to 0$. But, for special X_{10} of the form $X_{10} = X_9 \times S^1$, where X_9 is an arbitrary nine-manifold, one can perform a T-Duality transformation on the collapsing S^1 [8] so as to yield a theory with a non-singular

spacetime. We will examine this interesting property properly in section four. So, we will not discuss it further here.

3. Tensionless Strings : M-Theory on $S^1/\mathbf{Z}_2$

In this section we will examine a tensionless string appearing in M-Theory on $S^1/\mathbf{Z}_2$ from an $E_8 \times E_8$ Heterotic string theory perspective. We will then examine a tensionless string in the $E_8 \times E_8$ Heterotic string theory from a M-Theory perspective. Let us start, however, by examining the tensionless string in M-Theory on $S^1/\mathbf{Z}_2$ from an $E_8 \times E_8$ Heterotic string theory perspective.

3.1. Tensionless Strings in M-Theory on $S^1/\mathbf{Z}_2$

In this subsection we will examine a tensionless string in M-Theory on $S^1/\mathbf{Z}_2$ from an $E_8 \times E_8$ Heterotic string theory perspective. This examination, as we will find, is very similar to the Type IIA examination in the previous section. However, we will also find several critical differences. Let us now proceed with this examination.

M-Theory on $S^1/\mathbf{Z}_2$, as was proven by Witten and Horava [12], is equivalent to the $E_8 \times E_8$ Heterotic string theory. If we denote the radius of the $S^1/\mathbf{Z}_2$ factor as measured in the M-Theory metric by R_{11}, then one finds [3][10][12] that the ten-dimensional $E_8 \times E_8$ Heterotic string theory coupling constant $\lambda_{10,H}$ is given by,

$$\lambda_{10,H} = R_{11}^{3/2}. \tag{3.1}$$

In addition, one finds that the ten-dimensional $E_8 \times E_8$ Heterotic string theory metric $G_{10,H}$ and the ten-dimensional M-Theory metric $g_{10,M}$ are related [3][10][12],

$$G_{10,H} = R_{11} g_{10,M}. \tag{3.2}$$

Now, let us use this information to examine the appearance of a tensionless string in M-Theory on $S^1/\mathbf{Z}_2$.

M-Theory on $S^1/\mathbf{Z}_2$, as mentioned above, is equivalent to the $E_8 \times E_8$ Heterotic string theory. The $E_8 \times E_8$ Heterotic string theory obviously supports a one-brane. In the M-Theory picture this one-brane arises from a M-Theory two-brane wrapping about $S^1/\mathbf{Z}_2$. Hence, if we denote the M-Theory two-brane tension as $T_{2,M}$, then the tension of the one-brane in the M-Theory picture $T_{1,M}$ is given by [11],

$$T_{1,M} = T_{2,M} R_{11}. \tag{3.3}$$

So, if we consider the limit in which the one-brane tension, as measured in the M-Theory metric, goes to zero, then we see that in this limit $T_{2,M}R_{11} \to 0$. Now, in this case, as opposed to the Type IIA example of last section, we have a bit of freedom in interpreting this limit. The $\mathbf{Z}_2$ factor projects out the three-form of M-Theory [12]. Hence, it also projects out the M-Theory two-branes [4]. Thus, the quantity $T_{2,M}$ is not the tension of a two-brane which exists in M-Theory on $S^1/\mathbf{Z}_2$, but it is the tension of a two-brane which exists in the theory before the $\mathbf{Z}_2$ projection. So, to obtain the limit $T_{1,M} \to 0$ or equivalently $T_{2,M}R_{11} \to 0$, we can either take $R_{11} \to 0$ or $T_{2,M} \to 0$. Taking $T_{2,M} \to 0$, in contrast to the Type IIA case, does not correspond to taking the tension of a two-brane in M-Theory on $S^1/\mathbf{Z}_2$ to zero tension. It corresponds only to taking an "internal parameter" of the theory to zero. So, we are free to take $T_{2,M} \to 0$ or $R_{11} \to 0$ in both cases we only obtain a tensionless one-brane and no two-brane. Also, let us require that the M-Theory ten-metric $g_{10,M}$ be invariant in either limit, $g_{10,M} \to g_{10,M}$. In the limit $R_{11} \to 0$ the spacetime, from a M-Theory perspective, takes the form presented in Figure 3A.

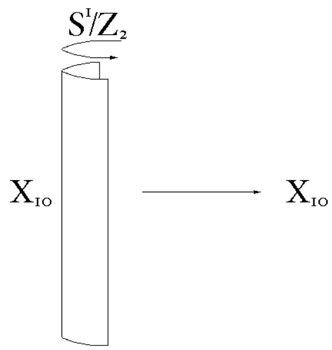

Figure 3A

Again, X_{10} is an arbitrary ten-manifold. However, if we consider the limit in which $T_{2,M} \to 0$, then the eleven-manifold does not become singular. In particular, $g_{10,M}$ and R_{11} are invariant in this limit. Let us now look at both of these limits from an $E_8 \times E_8$ Heterotic string theory perspective.

As mentioned in the previous section, a p-brane tension is a dimensionful quantity and thus depends upon the metric in which it is measured. Hence, the tension of the M-Theory one-brane is not the same when measured in the $E_8 \times E_8$ Heterotic string theory metric. So, in particular, employing the relation between the M-Theory ten-metric and the $E_8 \times E_8$ Heterotic string theory ten-metric (3.2) one finds that the one-brane tension $T_{1,H}$ as measured in the $E_8 \times E_8$ Heterotic string theory metric is given by,

$$T_{1,H} = R_{11}^{-1} T_{1,M} = T_{2,M}, \tag{3.4}$$

where the first equality follows from (3.2) and the second from (3.3).

So, from (3.4) one can see that in the limit $R_{11} \to 0$ the one-brane tension in the $E_8 \times E_8$ Heterotic string theory does not vanish. However, the ten-manifold X_{10} on which the Heterotic string theory resides does become singular. One can easily see this from equation (3.2) along with the fact that $g_{10,M} \to g_{10,M}$. Graphically, this is represented by Figure 3B.

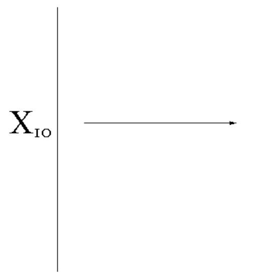

Figure 3B

As the limit $R_{11} \to 0$ is singular from an $E_8 \times E_8$ Heterotic string theory perspective, let consider our other option, the limit in which $T_{2,M} \to 0$.

In this limit, according to (3.4), one obtains a tensionless string in the $E_8 \times E_8$ Heterotic string theory. This, however, does not help our case as we do not know how to deal with the dynamics of such a string. But, as one can see from (3.2), the ten-manifold upon which the $E_8 \times E_8$ Heterotic string theory lives does not become singular in this limit. Graphically, this limit is represented in the same manner as depicted in Figure 2A. However, as there exists a tensionless string in this non-singular Heterotic theory, we do not know how to work with this theory in this $T_{2,M} \to 0$ limit. Now, let us consider reversing this process and looking at a tensionless string in the $E_8 \times E_8$ Heterotic string theory from a M-Theory point-of-view.

3.2. Tensionless Strings in the $E_8 \times E_8$ Heterotic String Theory

In this subsection we will examine the appearance of a tensionless string in the $E_8 \times E_8$ Heterotic string theory. We will then interpret this string in a M-Theory context employing the equality [12] of M-Theory on $S^1/\mathbf{Z}_2$ and the $E_8 \times E_8$ Heterotic string theory. Again, as most of the "leg-work" was performed in the previous subsection, we will refer to it heavily in this subsection.

According to equation (3.4) the one-brane tension $T_{1,H}$ as measured in the $E_8 \times E_8$ Heterotic string theory metric is $T_{1,H} = T_{2,M}$. Hence, the limit in which the $E_8 \times E_8$ Heterotic string becomes tensionless $T_{1,H} \to 0$ corresponds to the limit $T_{2,M} \to 0$. Furthermore, if we require that $G_{10,H} \to G_{10,H}$ in this limit , then the Heterotic limit is graphically represented by the situation in depicted Figure 2A. Now, let us look at the implications of this tensionless string from a M-Theory perspective.

As we showed above, $T_{2,M} \to 0$ in the limit $T_{1,H} \to 0$. However, according to equation (3.3) the one-brane tension in M-Theory is $T_{1,M} = T_{2,M} R_{11}$. So, the M-Theory one-brane tension not only depends upon $T_{2,M}$, but also upon R_{11}. Hence, we are left with a set of choices as to what we can take R_{11} to in this limit. If we require $T_{1,M}$ to be finite, then we must take R_{11} to infinity as $T_{2,M} \to 0$. This, while not yielding a singular $S^1/\mathbf{Z}_2$ from a M-Theory perspective yields a singular ten-manifold. This follows from (3.2) and the fact that $G_{10,H} \to G_{10,H}$. So, as we "generically" have no means to deal with M-Theory on such a manifold, let us consider some of the other possible limits.

If we allow $T_{1,M}$ to go to zero, then we can let R_{11} remain constant. This does not yield a singular $S^1/\mathbf{Z}_2$ or ten-manifold. However, it does yield a tensionless string in the M-Theory picture. Hence, as we do not know how to deal with such a tensionless string, this $T_{1,M}$ limit does not inform us in any substantial way. Finally, we may let $T_{1,M}$ go to zero while also taking R_{11} to zero. Again, this limit lands us in a theory about which little is known. As R_{11} goes to zero the $S^1/\mathbf{Z}_2$ factor is singular and little is known of M-Theory with $T_{1,M} = 0$ and $T_{2,M} = 0$ on such a manifold. Now, let us finally look at M-Theory on T^2. In many ways we will find this to be the most interesting case. Also, it is the one over which we will have the most control as we will be able to employ T-Duality to resolve some of the singular S^1's we will encounter.

4. Tensionless Strings : M-Theory on T^2

In this section we will examine the tensionless strings which appear in M-Theory on T^2. After this we will examine these tensionless strings from a Type II perspective. Let us now start this examination by looking at the tensionless strings in M-Theory on T^2.

M-Theory on T^2, as was proven by Witten [10], is equivalent to the Type IIA string theory on S^1. The Type IIA string theory on S^1 obviously possess a one-brane; in fact it possess two different one-branes. It possess the standard one-brane of Type IIA string

theory, but it also possess a second one-brane which arises from the two-brane of ten-dimensional Type IIA string theory wrapping about S^1. From a M-Theory perspective these two different one-branes have a similar origin.

Type IIA string theory on S^1 is equivalent to M-Theory on T^2 [10]. Hence, one could consider the M-Theory two-brane wrapping about either of the S^1's which "reside" in $T^2 = S^1 \times S^1$. Upon wrapping the M-Theory two-brane about the first S^1 one obtains the first one-brane. Upon wrapping the M-Theory two-brane about the second S^1 one obtains the second one-brane.

From a M-Theory point-of-view we can easily compute the tension of both these one-branes. If we denote the radius of the first S^1 as R_{10} and the radius of the second S^1 as R_{11}, both measured in the M-Theory metric, then the tension of the first one-brane ${T'}_{1,M}$ as measured in the M-Theory metric is

$${T'}_{1,M} = T_{2,M} R_{10}, \tag{4.1}$$

where $T_{2,M}$ is the tension of the M-Theory two-brane as measured in the M-Theory metric. Similarly, the tension of the second one-brane $T_{1,M}$ as measured in the M-Theory metric is

$$T_{1,M} = T_{2,M} R_{11}. \tag{4.2}$$

Now, one can easily see that to obtain a tensionless one-brane one can take $R_{10} \to 0$ or $R_{11} \to 0$. Also, one could take $T_{2,M} \to 0$ to obtain two tensionless one-branes. However, as we are interested only in understanding the appearance of a tensionless one-brane or one-branes in M-Theory, we will not consider the limit $T_{2,M} \to 0$ as it introduces, in addition to tensionless one-branes, a tensionless two-brane. Also, let us assume that in either of the two above limits the metric on the non-compact nine-manifold is invariant, i.e. $g_{9,M} \to g_{9,M}$. Let us now look at these limits, $R_{10} \to 0$ and $R_{11} \to 0$, from a Type II perspective.

Now, before we proceed in interpreting these tensionless strings in terms of a Type II string theory, we must first establish various relations between the variables of M-Theory and those of the Type II string theory. To some extent this was already done in section two. The ten-metric of the Type IIA string theory $G_{10,IIA}$, before compactification on S^1, is related to the ten-metric $g_{10,M}$ of M-Theory as follows,

$$G_{10,IIA} = R_{11} g_{10,M}. \tag{4.3}$$

Similarly, the ten-dimensional Type IIA coupling constant $\lambda_{10,IIA}$ is related to the S^1 radius R_{11} as follows,

$$\lambda_{10,IIA} = R_{11}^{3/2}. \tag{4.4}$$

Now, let us employ this information to interpret the tensionless strings appearing in M-Theory on T^2 in terms of a Type II theory.

Consider first the limit in which $R_{11} \to 0$. As we showed previously, the tension of the second one-brane as measured in the M-Theory metric is $T_{1,M} = T_{2,M} R_{11}$. So, as we are assuming $T_{2,M} \to T_{2,M}$, in this limit $T_{1,M} \to 0$. Similarly, upon looking at equation (4.1) one sees that $T'_{1,M} \to T'_{1,M}$ in this limit. So, we obtain a single tensionless string in the limit $R_{11} \to 0$.

Now, as we are assuming that $R_{10} \to R_{10}$ and $g_{9,M} \to g_{9,M}$ in this limit, the relation (4.3) implies that in the limit $R_{11} \to 0$ the ten-manifold[2] $X_9 \times S^1$ upon which the Type IIA theory is formulated is becoming singular. In particular, the metric $G_{10,IIA}$ for this ten-manifold is "vanishing." Hence, the interpretation of this particular limit from a Type IIA perspective does not seem to teach us much about the tensionless string in M-Theory on T^2. So, let us consider the second limit, that in which $R_{10} \to 0$.

In this limit, as one can easily see from equations (4.1) and (4.2) along with the assumption that $T_{2,M} \to T_{2,M}$, the tension $T'_{1,M}$ goes to zero and the tension $T_{1,M}$ is constant in this limit. Furthermore, as one can see from (4.3), the ten manifold $X_9 \times S^1$ does not collapse in the same manner as it did in the limit $R_{11} \to 0$. In the case at hand, the S^1 factor of $X_9 \times S^1$ goes to zero radius from a M-Theory perspective. Let us now consider what happens to the S^1 radius in the Type II picture.

Looking at (4.3) we can see that the ten-metric of the Type IIA theory is scaled relative to the M-Theory ten-metric. This relation implies that the radius R_{10} of the S^1 as measured in the M-Theory metric is different from the radius $R_{10,IIA}$ of the S^1 as measured in the Type IIA metric. In particular, (4.3) implies,

$$R_{10,IIA} = R_{11}^{1/2} R_{10}. \tag{4.5}$$

So, in the limit we are considering, $R_{10} \to 0$ and $R_{11} \to R_{11}$, the radius $R_{10,IIA}$ goes to zero, as it does in the M-Theory picture.

Now, at first this seems a little disappointing. It looks as if in the Type IIA theory we are again on a singular manifold as $R_{10,IIA} \to 0$. However, in this case we may employ

[2] Again, X_9 is an arbitrary nine-manifold.

T-Duality to resolve this limit. We can employ the standard T-Duality transformation [8] to interpret this limit in terms of a Type IIB string theory. According to Seiberg et. al. [8] Type IIA string theory on a S^1 with radius $R_{10,IIA}$ is equivalent to a Type IIB string theory on a S^1 with radius $R_{10,IIB} = 1/R_{10,IIA}$. In addition, the coupling constants of the theories are related by

$$\frac{R_{10,IIA}}{\lambda_{10,IIA}^2} = \frac{R_{10,IIB}}{\lambda_{10,IIB}^2}, \tag{4.6}$$

where $\lambda_{10,IIB}$ is the ten-dimensional coupling constant of the Type IIB string theory.

Now, in the limit $R_{10} \to 0$, as we found previously, $R_{10,IIA} \to 0$. This in turn implies that $R_{10,IIB} \to \infty$. Hence, the problem of the singular S^1 is solved if we consider instead the Type IIB theory on S^1. In this limit one finds that the tensionless string in M-Theory appears just as the S^1 on which the Type IIB theory is compactified becomes $\mathbf{R}$. So, we have found that in the limit in which one of the tensionless strings appears in M-Theory on $X_9 \times T^2$ M-Theory on $X_9 \times T^2$ is equivalent to the Type IIB string theory in ten-dimensions on $X_9 \times \mathbf{R}$.

This is indeed very interesting, however, there is a bit of a hitch. From equation (4.6), by employing the relation between the Type IIA and Type IIB radii along with the relation between the M-Theory radii and the Type IIA variables, one can compute the Type IIB coupling constant in ten-dimensions $\lambda_{10,IIB}$ in terms of the radii of M-Theory on T^2. One finds,

$$\lambda_{10,IIB} = R_{11}/R_{10}. \tag{4.7}$$

So, in the limit we are considering, $R_{10} \to 0$ and $R_{11} \to R_{11}$, the ten-dimensional Type IIB coupling constant goes to infinity. At first this looks to be a bit of a problem. But, upon a closer examination one find that this is indeed not the case.

The Type IIB string theory in ten-dimensions possess a $SL(2, \mathbf{Z})$ symmetry [6]. This symmetry, among other things, acts on the coupling constant of the Type IIB theory. In particular [1], it exchanges the weak and strong coupling regions of the theory. Hence, we can describe our above limit of a Type IIB string theory on $X_9 \times \mathbf{R}$ with $\lambda_{10,IIB} \to \infty$ by a Type IIB string theory on $X_9 \times \mathbf{R}$ with $\lambda_{10,IIB} \to 0$. In other words, we have found how to describe the appearance of a tensionless string in M-Theory on $X_9 \times T^2$ by a weakly coupled Type IIB string theory on $X_9 \times \mathbf{R}$.

However, as in all the other cases with which we have been dealing, there is a hitch. Consider the tension of the one-brane $T'_{1,M}$. In accord with our earlier remarks, it is given by $T'_{1,M} = T_{2,M} R_{10}$. Now, as the ten-dimensional Type IIA metric $G_{10,IIA}$ and the

ten-dimensional M-Theory metric $g_{10,M}$ are related as in equation (4.3), this implies that the tension of this one-brane $T'_{1,IIA}$ as measured in the Type IIA metric is

$$T'_{1,IIA} = R_{11}^{-1} T'_{1,M} = R_{10} R_{11}^{-1} T_{2,M}. \tag{4.8}$$

So, in the limit $R_{10} \to 0$, $R_{11} \to R_{11}$, and $T_{2,M} \to T_{2,M}$ the tension of this one-brane from a Type IIA perspective goes to zero. Now, in accord with T-Duality [1], the Type IIA metric $G_{9,IIA}$ on the nine-manifold X_9 is related to the Type IIB metric $G_{9,IIB}$ on the same nine-manifold by

$$G_{9,IIA} = G_{9,IIB}. \tag{4.9}$$

Hence, in particular, any tension measured in the Type IIA metric in nine-dimensions coincides with the same tension as measured in the Type IIB metric. So, in particular,

$$T'_{1,IIB} = T'_{1,IIA}, \tag{4.10}$$

where $T'_{1,IIB}$ is the tension of the one-brane $T'_{1,IIA}$ as measured in the Type IIB metric. Thus, as the tension $T'_{1,IIA} \to 0$ in the limit we are considering, so also $T'_{1,IIB} \to 0$ in the limit we are considering. Hence, the tension of the one-brane in the Type IIB theory on $X_9 \times \mathbf{R}$ goes to zero in the limit we are considering.

Thus, even though we are considering a weakly coupled Type IIB string theory on $X_9 \times \mathbf{R}$, we are also considering the limit in which[3] $T'_{1,IIB} \to 0$. Hence, as there is a tensionless string in the Type IIB spectrum, we actually do not know how to deal with this limit.

5. Generic Considerations

We have shown that many of the limits in which a tensionless string appears in M-Theory can be interpreted as singular limits of String Theory or limits in which a tensionless string appears in String Theory. However, one may also wonder if this suggests some general trend. One can relatively easily see that this is indeed the case.

Consider M-Theory compactified down to d dimensions on some manifold K. Let the d dimensional M-Theory metric be denoted by $g_{d,M}$. Also, let us assume that M-Theory

[3] Note, one may think that the $T'_{1,IIB} \to 0$ limit may be exchanged for another limit by way of $SL(2,\mathbf{Z})$; however, [7] this is not the case.

compactified down to d dimensions on K is equivalent to some string theory S let us call it. Let us denote the d dimensional string theory metric as $G_{d,S}$. Furthermore, let us assume that the two metrics are related by some scaling factor Q,

$$G_{d,S} = Q g_{d,M}. \tag{5.1}$$

Now, assume that there exists a one-brane in the M-Theory spectrum and its tension is denoted by $T_{1,M}$. Thus, the tension $T_{1,S}$ of this one-brane as measured in the string theory S is,

$$T_{1,S} = Q^{-1} T_{1,M}. \tag{5.2}$$

Now, let us consider the limit in which $T_{1,M} \to 0$ and $g_{d,M} \to g_{d,M}$. In this limit, if we wish to understand it from the perspective of the string theory S, we must require $T_{1,S} \to T_{1,S}$. So, this in turn implies $Q \to 0$. Now, as $g_{d,M} \to g_{d,M}$ and $Q \to 0$, we find that, in accord with (5.1), $G_{d,S} \to 0$. Hence, from a string theory perspective, if we wish to maintain a string of finite tension, we must have a singular d dimensional spacetime.

In addition, one could consider, instead of the limit in which $T_{1,S} \to T_{1,S}$, the limit in which we require $G_{d,S} \to G_{d,S}$, $g_{d,M} \to g_{d,M}$, and $T_{1,M} \to 0$. As $G_{d,S} \to G_{d,S}$ and $g_{d,M} \to g_{d,M}$ this implies, in accord with (5.1), that $Q \to Q$. So, in accord with (5.2), this implies that $T_{1,S} \to 0$ as a result of $T_{1,M} \to 0$. So, in this case we do not obtain a singular d dimensional spacetime from a string theory perspective, but we do obtain a tensionless string with which we do not know how to work.

So, it seems this is a general phenomena. The appearance of a tensionless string in M-Theory on a non-singular spacetime can be interpreted as the appearance of a tensionless string in string theory on a non-singular spacetime. Or it can be interpreted as the collapse of spacetime with a string of finite tension from a string theory perspective.

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